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ABSTRACT

This paper reanalyzes the Hansen-Weisbrod-Pechman debate concerning the equity effects of subsidized tuition for higher education. It presents an intergeneration model of income distribution, including educational effects. This model is used to analyze issues in the debate over equity effects. It yields a measure (equity measure) of the sensitivity of income inequality to the level of higher educational subsidy. The equity measure represents what cannot be estimated directly due to lack of data. The model also yields a net benefit measure which represents what can be estimated from available data. The main conclusion from the model is a negative one. In a simple economy, net benefit calculations do not provide a reliable indicator of the equity effects of tuition subsidies for higher education. It is not clear how this conclusion could be reversed by complicating the economy. The conclusion is a negative one because it indicates that the current feasible procedure for answering an important policy question is not reliable. Nevertheless, the model does provide a formal structure, previously lacking, for analyzing issues in this debate. (Author/AM)

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A FURTHER LOOK AT THE HANSEN-WEISBROD-PECHMAN DEBATE

John Conlisk

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A FURTHER LOOK AT THE HANSEN-WEISBROD-PECHMAN DEBATE*

JOHN CONLISK

ABSTRACT

An intergenerational model of the income distribution, including educational effects, is presented. It is used to analyze issues in the debate over equity effects of subsidies for higher education. In particular, the analysis supports Pechman's view that conventional net benefit calculations are poor indicators of such equity effects.

In this *Journal* [4, 7, 8, 9, 11, 13, 14] and elsewhere [16, references there], Hansen and Weisbrod, Pechman, and others have debated the equity effects of subsidized tuition for higher education. At the heart of the debate are net benefit calculations presented by Hansen and Weisbrod [7]. Pechman's critical view of these calculations may be summarized in his own words [16, p. S256]:

> The traditional way of looking at the burdens and benefits of higher education is to distribute the net benefits received by students by the income classes of parents (taxpayers). This sweeps the problem created by the intergenerational nature of the benefit transfer under the rug. It seems to me that a more useful way of looking at the problem is to acknowledge that the benefits of public higher education are received by one generation while the costs are paid by another and that there is no way of merging the benefits and costs in one distribution to evaluate the equity of the system.

If such net benefit calculations do not measure the equity effects of subsidized tuition, what calculations do? Intuitively, the issue involves comparison of two societies identical in all respects except that one subsidizes higher education more generously. As time passes will a difference in distributional

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equality develop between the societies, and in which direction? Analytically, a natural approach to this question is to construct a complete intergenerational model of the size distribution of income, including higher education variables and tuition subsidy parameters; then to derive income distribution indicators and analyze the sensitivities of the indicators to changes in the subsidy parameters, and, in particular, to see whether distributional inequality increases or decreases with an increase in the rate of subsidization.

Sections I and II present an appropriate intergenerational model. In principle, a sufficiently elaborated version of the model, fit to appropriate data, might yield a respectable estimate of the equity effects of higher educational subsidies. However, the appropriate data are not available, and the needed elaborations of the model would be major projects in themselves. Therefore, this paper sets two more modest goals. The first is to propose a promising approach for analyzing the subsidization issue. The second is to use the model in sorting out the logic of the net benefit calculations referred to in the Pechman quotation.

Sections III and IV concern the second goal. The model will yield a measure—call it the equity measure—of the sensitivity of income inequality to the level of higher educational subsidy. The equity measure will represent what we would like to know, but cannot estimate directly due to lack of data. The model will also yield a net benefit measure of the sort referred to in the Pechman quotation. The net benefit measure will represent what we can directly estimate from available data (as Hansen and Weisbrod [7] have done). The Hansen-Weisbrod-Pechman debate will be interpreted as a debate over the following proposal: that the net benefit measure be used as a proxy for the equity measure. Hansen and Weisbrod might be identified with the affirmative, Pechman with the negative. It turns out in the model that the equity measure is not well proxied by the net benefit measure, thus favoring Pechman's side.

Of course, it should be emphasized that the true debate (as opposed to my characterization of it) is about equity and net benefits in real life, not in a highly simplified model. However, if the logical connection between equity and net benefits does not hold up in a simple economy, it seems improbable that it will hold up in a complicated one.

I. THE MODEL

The model may be informally described as follows: Each individual of each generation has schooling and income variables that are determined by a simple simultaneous equations system. The parameters of the system are the same for every individual in every generation. However, there are stochastic disturbance terms appended to the equations, and they are generated separately and independently for each individual of each generation. They cause individual differ-

ences in the schooling and income variables, and they represent the great array of random events that make individuals unlike in real life. In addition, the equations determining an individual's variables contain the same variables defined on that individual's parent. The parental variables represent real-life forces leading to parent-child similarities. As the Pechman quotation emphasizes, the intergenerational dynamics are central to the debate. Linearity and other assumptions are introduced to make the model tractable. Turn now to a formal description.

Assume that individuals are unisexual and that each has one offspring, so population is constant and family trees are simple straight lines. Individuals attain the same legislated amount of "lower" schooling and variable amounts of "higher" schooling. Interest centers on the sensitivity of distributional measures to the level of tuition charged for higher schooling. The finance of lower schooling is left behind the scenes. The time unit is a generation. Let $Y_i(t)$, $S_i(t)$, $X_i(t)$, and $T_i(t)$ be the before-tax lifetime income, amount of higher schooling, after-tax lifetime income, and lifetime tax of the tth generation individual of the ith family tree. Thus, the values of these variables for this individual's offspring are $Y_i(t+1)$, $S_i(t+1)$, $X_i(t+1)$, and $T_i(t+1)$. Assume the values of these variables, for all i and for $t = 1, 2, \ldots$, are determined by the linear stochastic difference equation system:

$$(1) \qquad Y_i(t+1) = \alpha + \beta S_i(t+1) + \gamma[X_i(t) - PS_i(t+1)] + \delta S_i(t) + u_i(t)$$

$$(2) \qquad S_i(t+1) = f(P) + h(P)X_i(t) + bS_i(t) + v_i(t)$$

$$(3) \qquad X_i(t) = Y_i(t) - T_i(t)$$

$$(4) \qquad T_i(t) = -G + H(t)Y_i(t)$$

$$(5) \qquad T(t) = gY(t) + (C - P)S(t+1)$$

Equation (1) states that an offspring's lifetime income $Y_i(t+1)$ is a linear function of his schooling $S_i(t+1)$ and of his parent's characteristics $X_i(t) - PS_i(t+1)$ and $S_i(t)$, plus a disturbance term $u_i(t)$. Here P is the tuition price of higher schooling, so $X_i(t) - PS_i(t+1)$ is the amount of lifetime income the parent has left after paying his tax and his offspring's tuition. This income, together with the parent's schooling level $S_i(t)$, may be thought of as influencing the richness of the home environment in which the offspring develops his capabilities, as well as influencing any direct financial inheritance. The natural assumptions $\beta > 0$, $\gamma \geq 0$, and $\delta \geq 0$ are made. The disturbance terms $u_i(t)$ in (1) and $v_i(t)$ in (2) will be the ultimate sources of distributional inequality in the model. The pair $[u_i(t), v_i(t)]$ is assumed to have zero mean and to have variances $(\sigma_{uu}, \sigma_{vv})$ and covariance σ_{uv} constant over all i and t. Also, the pairs $[u_i(t), v_i(t)]$ are assumed independent over all i and t.

Equation (2) states that the amount of schooling received by an offspring is a linear function of his parent's after-tax income and schooling, plus the disturbance term $v_i(t)$. The constant term $f(P)$ and the coefficient $h(P)$ are functions of the tuition parameter P; for simplicity, neither the coefficient b nor the shock variance σ_{vv} depends on P. Equation (2) may be thought of as a demand for schooling function. The natural assumptions $h(P) \geq 0$, $b \geq 0$, and $f'(P) + h'(P)X_i(t) < 0$ over the relevant $X_i(t)$ range are made. In the numerical calculations below, $f(P)$ and $h(P)$ will be specified as linear functions of P, so (2) will take the form

$$S_i(t+1) = a_0 + a_1P + a_2X_i(t) + a_3PX_i(t) + bS_i(t) + v_i(t) \tag{6}$$

Since the price derivative here is $a_1 + a_3X_i(t) < 0$, the parameter a_3 determines whether a tuition change influences low-income students more ($a_3 > 0$) or less ($a_3 < 0$) than high-income students.

Equation (3) is definitional. Equation (4) is the individual income tax function. It has a negative range if $G > 0$, where G interprets as the guaranteed lifetime income. The tax rate $H(t)$ varies with time as necessary to satisfy the government's budget constraint, which is equation (5). In (5) and in the analysis to follow, the sample mean of a subscripted variable is denoted by the corresponding unsubscripted symbol:

$$Y(t) = \Sigma_i Y_i(t)/N \qquad S(t) = \Sigma_i S_i(t)/N \tag{7}$$

$$X(t) = \Sigma_i X_i(t)/N \qquad T(t) = \Sigma_i T_i(t)/N$$

where N is the population size. The government is assumed to spend a fraction g of national income on public goods and services (including lower but not higher schooling). The cost of one unit of higher schooling is denoted C. Thus, the budget constraint (5) says (in per capita terms) that the tax revenue $T(t)$ of a parental generation must equal the public goods and services expenditure $gY(t)$ of the generation plus the higher schooling bill $(C - P)S(t+1)$ for the generation's offsprings.

It is assumed that the population N is large enough that the sample means (7) may be treated as nonstochastic with negligible loss of accuracy. In view of the size of real-life populations, this assumption seems innocent, and it greatly simplifies analysis.

II. DISCUSSION OF THE MODEL

Given the value of income $Y_i(t)$ and schooling $S_i(t)$ for every individual of an initial generation, and given the joint density function of the disturbance pair $[u_i(t), v_i(t)]$, the model determines the time paths of all variables over all future

generations. Consider in step-by-step terms how the transition from generation t to generation $t+1$ takes place:

1. We start with $Y_i(t)$ and $S_i(t)$ for all i.

2. Mother Nature generates disturbance term values $u_i(t)$ and $v_i(t)$ for all i.

3. Equations (2) to (5) now solve simultaneously for the tax rate $H(t)$ and the variables $[S_i(t+1), X_i(t), T_i(t)]$ for $i = 1, \ldots, N$. There are $3N+1$ variables here and the same number of equations; each of (2) to (4) contributes N equations. [$Y(t)$, $T(t)$, and $S(t+1)$ in (5) are not additional variables; they just denote averages.] Intuitively, the reason the equations must be solved simultaneously is this: The offspring generation schooling variables $S_i(t+1)$ depend on the parental generation after-tax incomes $X_i(t)$. But after-tax incomes $X_i(t)$ in turn depend on how much schooling $S_i(t+1)$ the parents support through taxes.

4. Equation (1) determines $Y_i(t+1)$ for all i. We now have $Y_i(t+1)$ and $S_i(t+1)$ for all i.

In summary, the model determines the evolution over generations of the joint distribution over families of the endogenous variable vector $[Y_i(t), S_i(t), X_i(t), T_i(t)]$.

The linearity of the model in i-subscripted variables allows a derivation of simple expressions for the mean and variance matrix of $[Y_i(t), S_i(t), X_i(t), T_i(t)]$, both in and out of equilibrium. The expressions involve the disturbance variance parameters ($\sigma_{uu}, \sigma_{vv}, \sigma_{uv}$), but the exact distribution of $[u_i(t), v_i(t)]$ is not needed. The mean and variance matrix of $[Y_i(t), S_i(t), X_i(t), T_i(t)]$, while far short of the whole distribution, provide enough information to compute coefficient of variation measures of distributional inequality and parent-child correlation measures of social immobility. What follows concentrates on these distributional measures, especially their sensitivity to the tuition parameter P. The derivation of the expressions is contained in the Appendix. Since the expressions are tedious to state and impossible to interpret by sight, their formal statement will be left in the Appendix. Certain stability conditions described in the Appendix are assumed throughout.

Intuitively, the model's behavior may be thought of in regression-to-the-mean terms. If the disturbances were suppressed, all incomes would gradually converge to the same value (regress to the mean). However, the presence of the disturbance terms prevents this outcome. Equilibrium in the model is a balance between the equalizing force of the regression-to-the-mean mechanism and the disequalizing force of the disturbance.

The ingredients of the model are familiar in social science, but the combinations may not be. Thus, it may be helpful to relate this modelling approach to three more familiar contexts.

Macroeconomic Modelling. The usual macroeconomic model determines aggregate income and other variables through a system of simultaneous equations. Different models have different equations, but the common analytic

framework unifies and promotes progress in the whole field. The strategy of the modelling here is to exploit the same flexible and unifying framework in the context of distributional phenomena. The main complicating factors are the presence of an entire population of observational units (instead of just one as in a macro model) and the central importance of the disturbance terms (which are often suppressed in a macro context).

Socioeconomic Achievement Models. Sociologists and economists have done much recent modelling in which an individual's characteristics (schooling, occupational status, IQ, and so on) are determined by a system of equations [6 and references there]. These models contain disturbance terms and parental variables, so they implicitly have the intergenerational dynamic structure of the model here. However, the distributional dynamics of these models are not worked out. Attention focuses rather on estimation of the coefficients and what the estimates imply about a single individual's behavior.

Discrete Markov Chains. In the substantial Markov chain literature on intergenerational mobility (reviewed in [1]), dynamic and equilibrium distributional properties are heavily emphasized. The model here is very much in the spirit of these Markov chain models. The problem with the Markov chain approach is that continuous variables must be forced into discrete categorizations, and the number of parameters soon gets out of hand. In the model here, there are four variables defined on each individual. If each variable were described by a five-way categorization, there would be $5^4 = 625$ possible states an individual could be in, and the transition matrix of the Markov chain would have $625(624) = 390{,}000$ independent elements. The model here is similar in spirit to a Markov chain, but formulated in continuous variable terms to economize on parameters.

A feature of the usual socioeconomic achievement and Markov chain approaches is that individual behaviors do not interact within a generation; individuals of a generation are modelled in isolation of each other. In a Markov chain, for example, the transition probabilities applying to an individual do not depend on the distribution of other individuals over states. In contrast, the model here, through the aggregate budget constraint (5), accounts for a type of individual interaction that is crucial to the context. (The fact that educational subsidies have to be paid for is central to the equity issue.) It is an advantage of the modelling structure here that such interaction can be easily accommodated.

By adding or subtracting equations and terms within equations, models of the same general structure as the one here can be tailored to a variety of distributional contexts. And various generalizations of the structure can be accommodated while retaining analytic tractability—capital accumulation and economic growth, two sexes, assortative mating, variable fertility, subperiods within generations, and others. On the negative side, the restrictive linearity assumption is necessary for analytic results. (Such issues are discussed in working papers available on request. An application of the modelling approach to a different issue is in [5].)

III. EQUITY AND NET BENEFITS

We now have a complete intergenerational model, where distributional magnitudes are expressed (in the Appendix) as functions of the model's parameters and initial conditions. The natural question is how distributional inequality changes as the tuition parameter P changes. Let us focus on inequality in after-tax lifetime income $X_i(t)$, as measured by its coefficient of variation, call it $V_X(t)$:

$$V_X(t) = [\text{Standard deviation of } X_i(t)] / [\text{Mean of } X_i(t)] \tag{8}$$

How does the path of $V_X(t)$ change when P changes? The answer turns out to be thoroughly ambiguous.

First, consider the ambiguity at an intuitive level. (The intuitive discussion will be deliberately nonspecific about which generation after the P-change is being discussed; the ambiguity applies to them all.) Start with the sensitivity of the numerator of (8) to P. A proponent of subsidies to higher schooling might argue that an increase in P would close off an avenue of advancement to children of low-income parents, while children of high-income parents would be little influenced. It is easy to specify parameters in the schooling-demand function (2) and the other functions, so that this mechanism is operative and dominant. That is, the sensitivity of $\partial \text{Var}[X_i(t)] / \partial P > 0$ can occur. An opponent of subsidies to higher schooling might argue that an increase in P would hurt children of lower-income parents very little, since few of them attain higher schooling, whereas the tax cut made possible by a P-increase would enable low-income parents to make other advantages, already available to children of the rich, available to their children. [This "other advantages" mechanism works through the γ-term in (1).] It is also easy to find parameter values that illustrate this view; that is, $\partial \text{Var}[X_i(t)] / \partial P < 0$ can occur.

Thus, if the variance of $X_i(t)$ were the chosen inequality measure, the equity effect of a change in P would be ambiguous. However, the coefficient of variation $V_X(t)$ is a more conventional inequality measure. To compound the ambiguity, the denominator of $V_X(t)$ [the mean of $X_i(t)$] may also move either up or down in response to an increase in P (regardless of which way the numerator moves). A proponent of subsidies to higher schooling might argue that an increase in P, by depressing average school attainment, would depress average income, thus promoting an increase in inequality $V_X(t)$ through a decrease in its denominator. This pattern can occur in the model. On the other hand, an opponent of higher schooling subsidies might argue that an increase in P, though reducing average schooling attainment, might nonetheless increase average after-tax income, both by reducing taxes and by freeing resources that increase before-tax income through nonschooling mechanisms [represented in (1) by the γ-term]. And an increase in average after-tax income would help to reduce inequality $V_X(t)$. This pattern may also occur in the model.

In short, there is little reason to hope for neat qualitative results about the sensitivity of the $V_X(t)$ path to P. And indeed I have found no simple sufficient conditions on parameters for signing $\partial V_X(t)/\partial P$ (except trivial ones that assume away the issues). To complicate the problem, the data needed to estimate the complete model are not available (as discussed in the next section). Since qualitative analysis fails and data are unavailable, we seem to be stuck. However, into this quandary comes the idea of net benefit calculations.

The idea of a net benefit calculation is to break up the population into income classes and then to see whether each class gains more or less in subsidy benefits than it pays out in taxes to support the higher schooling system. If lower-income classes get positive net benefits, while higher-income classes get negative net benefits, it appears to indicate that higher schooling subsidies promote equity, and vice versa. Since net benefit data are available, we have an operational proposal for discovering at least the direction in which a P-change pushes inequality.

As indicated in the Pechman quotation, the problem with the net benefit approach is the ambiguity of breaking down the population by income class. For a given generation, each observational unit in the population consists of a child receiving benefits and a parent paying taxes. Whose income class applies, the parent's class or the child's future class? In practice, the parent's class is chosen. If society were so rigidly stratified that the two classes were identical, or very close, then the ambiguity would not be serious. However, if parents' and children's income classes are only weakly correlated, then the ambiguity can lead to net benefit measures that give the wrong answer. Consider an example:

Suppose net benefits tend to increase with income level (children of higher-income parents tend to receive subsidies that exceed their parents' schooling taxes). This suggests that the subsidies promote inequality. How might the opposite be true? If child's income is only weakly correlated with parent's income, and if schooling is only weakly correlated with income, then a child's subsidy benefit may be only weakly correlated with his *own* adult income. The tax-subsidy system would then have the following two characteristics: (1) Taxes would be collected from individuals of each generation (as parents) strictly according to their incomes. (2) Subsidies would be dispersed to individuals of each generation (as students) largely at random relative to their (later adult) incomes. This description suggests that the system may well reduce inequality. The next section presents examples of such an outcome. The next section also cites evidence suggesting that parent-child correlations and schooling-income correlations are indeed weak.

To lay a foundation for the next section, a formal definition of the net benefit proposal is needed. Suppose the model is in equilibrium as of the beginning of generation $t = 0$, but that the tuition level P undergoes a permanent change ΔP beginning with the children of generation $t = 0$ parents (so these parents have to pay the new tuition level for their children's higher schooling).

The impact on inequality may be measured by $\Delta V_X(t)/\Delta P$ ($t = 1, 2, \ldots$), where $\Delta V_X(t)$ is the difference between $V_X(t)$ and what it would have been in the absence of the P-change (namely, the original equilibrium value). The net benefit proposal will be interpreted as the sign equality

$$\text{(9)} \qquad \text{sign}\,[\Delta V_X(t)/\Delta P] = -\,\text{sign}(Z) \quad \text{for } t = 1, 2, \ldots,$$

where Z (defined below) is a net benefit index measured on the initial equilibrium. In words, (9) says that the direction of the net benefit pattern prevailing before a tuition change predicts the direction of the equity effect of that change.

To construct the equilibrium net benefit index Z, which is measured on society as a whole, we first need a measure, call it $M_i(t)$, of the net benefit of tuition subsidies to the ith individual family of generation t. There is considerable arbitrariness in deciding how many of a parent's tax dollars go to pay for higher schooling. One possibility, adopted here, is to assume that the fraction of any parent's tax bill going to higher schooling is the same as the fraction of society's total tax bill going to higher schooling. In equilibrium, this fraction is $(C - P)S/T$. [Variable names with the time index dropped denote equilibrium values, so H is the equilibrium marginal tax rate and (Y, S, X, T) are equilibrium means.] Then the net benefit when the model is in equilibrium for the tth generation of the ith family tree is

$$\text{(10)} \qquad M_i(t) = \left(\begin{matrix}\text{higher schooling}\\ \text{subsidy for offspring}\end{matrix}\right) - \left(\begin{matrix}\text{higher schooling}\\ \text{tax on parent}\end{matrix}\right)$$

$$= (C - P)S_i(t+1) - (C - P)(S/T)T_i(t)$$

$$= (C - P)\,[(GS/T) + S_i(t+1) - (HS/T)Y_i(t)]$$

The conventional net benefit approach, represented in terms of the model, is to compute the conditional expectation $E[M_i(t)|Y_i(t)]$ of the net benefit $M_i(t)$ for various levels of income $Y_i(t)$. This expectation depends on $E[S_i(t+1)|Y_i(t)]$. Assuming linearity of conditional expectation, the function $E[S_i(t+1)|Y_i(t)]$ is given in equilibrium by the equilibrium population equation

$$\text{(11)} \qquad E[S_i(t+1)|Y_i(t)] = S + r_{+SY}(S/Y)(V_S/V_Y)\,[Y_i(t) - Y]$$

Here r_{+SY} denotes the equilibrium correlation of child's schooling with parent's income, and subscripted Vs denote coefficients of variation (formulas for these magnitudes are in the Appendix). Substituting (11) into the conditional expectation of (1) yields

$$E[M_i(t)|Y_i(t)] = (\text{term not depending on } i) + ZY_i(t)$$

where Z is defined as

$$Z = -(C - P)(S/Y)\,[(HY/T) - (r_{+SY}V_S/V_Y)]$$

Since $E[M_i(t)|Y_i(t)]$ is linear in $Y_i(t)$, the relation of net benefits to income level can be measured by a single number, the slope Z. That is, Z measures the degree to which a schedule of net benefits by income level would rise with income level. Z in the net benefit proposal (9) is assumed to be evaluated at the model's equilibrium prior to the change in P.

There is no question concerning equality (9) as a strict mathematical theorem. It is false for some parameter specifications, as the numbers in the next section will show. However, from a practical viewpoint, the relevant question is whether, over a variety of parameter specifications, equality (9) "usually" gives the right answer. The next section addresses this question.

IV. COUNTEREXAMPLES

It would be a routine econometric exercise to estimate the model, given appropriate data. However, matched parent-child lifetime income data are not available, or close to it. Matched parent-child current income data are virtually nonexistent, as are lifetime income data of any kind. Further, a serious estimate of the model, to avoid specification bias, would require elaboration of the model to account for other phenomena known to be important (growth in average income, explicit treatment of wealth and property income, and so on). And these elaborations would lead to further data obstacles.

Therefore, an empirical implementation of the model is ruled out. Nonetheless, it is possible to do something quite useful with manufactured numbers, namely, to test the logic of the net benefit proposal (9). The values of Z and $\Delta V_X(t)/\Delta P$ can be computed for a variety of parameter specifications. We can then see if positive (negative) values of $\Delta V_X(1)/\Delta P$, $\Delta V_X(2)/\Delta P$, . . . are typically associated with negative (positive) values of Z. If so, we will have at least some reason to believe that net benefit measures might provide useful evidence on equity effects. If not, we will have found no evidence to dispute Pechman's contrary assertion.

In manufacturing points in the parameter space, I will use what scraps of evidence I know of to locate in a reasonable region, though it is worth repeating that what follows is not an empirical study in any usual sense of the term.

Suppose each individual completes 12 years of lower schooling at age 17. Let $S_i(t)$ be defined as the present value as of age 17 of the costs of the individual's higher schooling beyond age 17, including forgone earnings. Let $Y_i(t)$ be measured as the present value as of age 17 of the individual's lifetime income stream. In this context, the following specification was chosen as a bench mark, where $f(P)$ and $h(P)$ are as in (6). The next several paragraphs explain.

(12) $\quad \alpha = 96, \beta = 1, \gamma = .22, \delta = .22,$

$\quad a_0 = 4.5, a_1 = -.36, a_2 = .16, a_3 = 0, b = .29,$

$$G = 0, g = .25, C = .4, P = .14,$$

$$\sigma_{uu} = 3900, \sigma_{vv} = 1300, \sigma_{uv} = -300$$

Parameter values were chosen partly by specifying values directly from scraps of evidence and partly by trial-and-error manipulation of parameter values until the equilibrium implied by (12) appeared consistent with other scraps of evidence. Thus, we need to have the equilibrium implied by (12) in front of us:

(13)
$$(Y, S, X, T) = (145, 23, 103, 42), H = .29,$$

$$(V_Y, V_S, V_X, V_T) = (.52, 1.78, .52, .52),$$

$$(r_{+YY}, r_{+SS}, r_{+XX}, r_{+TT}) = (.40, .40, .40, .40), r_{YS} = .52$$

The subscripted Vs are the equilibrium coefficient of variation inequality measures for the indicated variables. The subscripted r_+s are equilibrium parent-child correlations (measures of social immobility) for the indicated variables; and r_{YS} is the equilibrium contemporaneous income-schooling correlation.

Let us proceed from the more to the less straightforward specifications. The specification $g = .25$ says that government goods and services are one fourth of the national income. The specification $G = 0$ makes the income tax proportional. This is a reasonable first approximation to reality [12], and it has the added advantage in this context of not confounding redistributive tuition subsidy effects with redistributive negative tax effects. By definition, C is the fraction of the unit schooling cost represented by direct schooling costs (as opposed to forgone earnings); the value $C = .4$ is consistent with figures in Schultz [15, Table 1]. Tuition, of course, varies tremendously across real-life colleges. The specification $P = .14$ asks us to consider an economy in which all students pay tuition covering 35 percent of direct costs. The specification $\beta = 1$ implies [see (1)] that one dollar is the expected present value pretax payoff to an individual of an additional present value dollar of schooling investment in him, which seems plausible for a society in equilibrium.

In the schooling demand function (6), the parameter a_3 equals the cross partial $a_3 = \partial^2 S_i(t+1)/\partial P \partial X_i(t)$. In the absence of empirical evidence or prior reason for arguing a particular sign, $a_3 = 0$ was chosen for the bench mark specification. Given $a_3 = 0$, $a_2 = \partial S_i(t+1)/\partial P$ is a measure of the price sensitivity of schooling demand. Empirical results in [3] and [10] suggest a price elasticity of schooling demand in the $-.5$ vicinity. Given the constant elasticity demand function used in these studies, a $-.5$ elasticity implies that a P-increase from $P = .14$ to $P = C = .4$ will reduce S to $(.4/.14)^{-.5}$ times (about 60 percent less than) its original value. The value $a_1 = -36$ in the linear function (6) was chosen because, for the same change in P, it also implies about a 60 percent schooling demand reduction (relative to the equilibrium mean value $S = 23$).

Inspection of (1) and (6) shows that the coefficients (γ, δ, a_2, b) of the lagged (parental) variables are the crucial ones in determining the strengths of the equilibrium parent-child correlations in (13). (For intuition here, recall that, in the single equation model $y_{t+1} = \rho y_t + u_t$ with white noise error u_t, the equilibrium lag correlation is ρ.) Since reported parent-child schooling and occupational status correlations tend to run in the .35 to .45 vicinity [6, various tables], it seemed reasonable to put r_{+YY} and r_{+SS} in this vicinity, as they are in (13). This decision gave two pieces of information crucial to setting the four parameters (γ, δ, a_2, b). In the absence of evidence on how to proceed further, it was arbitrarily decided to set γ and δ so that their corresponding beta-coefficients in (1) are of comparable size, and similarly for a_2 and b in (6). These two additional decisions, which are reflected in (12) and (13), fairly well settled (γ, δ, a_2, b), given the other specifications.

The constant terms a and a_0, given the other specifications, were chosen mainly to yield sensible equilibrium means for income and schooling. The dollar magnitudes (Y, S, X, T) in (13) were assumed to be in thousands, discounted to present value terms at rate .08. The average lifetime income value $Y = 145$ is, for example, roughly consistent with a 47-year working lifetime, after two years of schooling beyond age 17, at an annual income of $14 thousand. (That is, 145 approximately equals $14 \sum_{t=3}^{49} 1.08^{-t}$.) The value $S = 23$ is roughly consistent with an average of two years of schooling beyond age 17 at an annual cost of $7.5 thousand forgone earnings plus $5 thousand direct schooling costs, discounted back to age 17.

Finally, the disturbance variance parameters $(\sigma_{uu}, \sigma_{vv}, \sigma_{uv})$ were specified, given other parameters, to yield plausible coefficient of variation inequality measures in (13) and to yield a plausible income-schooling correlation r_{YS} in (13). Despite the mass of current income data available, very little is known about the degree of inequality in lifetime incomes. Blinder [2, pp. 102-104], after reviewing the small amount of panel data evidence he was able to find, concludes that the Gini measure of lifetime income inequality may be in the .25 to .30 range. Assuming a lognormal distribution, this translates to a .47 to .59 range for the coefficient of variation, which agrees with V_Y in (13). In the model, the coefficient of variation V_S is defined on schooling beyond 12 years, so V_S should be much larger than if it were defined on all schooling. The V_S value in (13) is consistent with reported (age-specific) distributions of years of post-secondary schooling attained [17, Table 1]. The parameters σ_{uu} and σ_{vv} were the crucial ones in determining V_Y and V_S (given other parameters), while σ_{uv} was manipulated mainly to produce a reasonable r_{YS}. There are no reported correlations of lifetime income with schooling, but correlations of occupational status with schooling tend to run in the .5 to .6 vicinity, while correlations of current income with schooling run somewhat lower [6, various tables]. Thus, the r_{YS} value in (13) seems reasonable.

Suppose the tuition level is changed from $P = .14$ to $P = 0$ starting with the

generation of $t = 0$ parents, and assume the model was in equilibrium prior to this change. For the bench mark specification, $Z = .010$ prior to the P-change and the $\Delta V_X(t)/\Delta P$ are:

t	1	2	3	...	∞
$\Delta V_X(t)/\Delta P$	.085	.157	.193	...	.229

The positivity of Z implies that the net benefits of higher schooling subsidization rise with income level, suggesting that the tuition decrease will increase inequality. In fact, however, the $\Delta V_X(t)/\Delta P$ values are positive. That is, the bench mark specification is itself a counterexample to the net benefit proposal (9).

For the bench mark specification, sign $[\Delta V_X(t)/\Delta P]$ is the same for $t = 1, 2, \ldots$. This is also true for most other specifications considered. Therefore, little is lost by collapsing the net benefit proposal (9) to the single sign equality $\text{sign}(\Delta V_X/\Delta P) = -\text{sign}(Z)$, where ΔV_X denotes the full old-to-new equilibrium change.

To compare sign $(\Delta V_X/\Delta P)$ to sign (Z) over a variety of alternate specifications, the following low and high [relative to (12)] alternate values were chosen for seven of the parameters:

	β	γ	δ	a_1	a_2	a_3	b
Low	.5	.10	.10	-50	.10	-.5	.15
High	1.5	.35	.35	-20	.25	.5	.45

All possible combinations of these values, holding other parameters fixed at values (12), were used as alternate specifications. There are $2^7 = 128$ such specifications. Categorizing them by the signs of Z and $\Delta V_X/\Delta P$ yields the thoroughly mixed pattern:

	$\Delta V_X/\Delta P < 0$	$\Delta V_X/\Delta P > 0$
$Z < 0$	19	13
$Z > 0$	29	67

The upper left and lower right cells represent counterexamples to the net benefit proposal sign $(\Delta V_X/\Delta P) = -\text{sign}(Z)$. There are 86 counterexamples out of 128 cases. Recomputing the table for G-specifications other than $G = 0$ (thus allowing income tax progressivity or regressivity), or for alternate P-changes, often leads to quite different sign patterns. However, in all such recomputations, there were sizable numbers of counterexamples. Furthermore, there didn't appear to be any simple characterization of parameter combinations for which the net benefit proposal is reliable.

These numbers indicate two things. First, the net benefit proposal is cer-

tainly not correct as a theorem, since one counterexample is adequate disproof. Second, the net benefit proposal appears to be highly unreliable even as an approximation, since the counterexamples abound. On the second point, however, it should be emphasized that the numbers are manufactured and thus might be eccentric in some crucial sense.

V. CONCLUSION

The main conclusion is a negative one. In a simple economy, net benefit calculations do not provide a reliable indicator of the equity effects of tuition subsidies for higher education. It is not clear how this conclusion could be reversed by complicating the economy. In terms of the Hansen-Weisbrod-Pechman debate, the conclusion favors Pechman's side. It is a negative conclusion because it tells us that a currently feasible procedure for answering an important policy question is not reliable.

The paper also has a constructive side. The model provides a formal structure, previously lacking, for analyzing issues in the debate. If net benefit calculations will not answer our questions, the model suggests the calculations that will. Furthermore, the model is eminently tractable econometrically. This tractability is little comfort so long as the data are inadequate. However, the data are improving rapidly.

APPENDIX

Under the assumption that N is large, the shock term sample means $\Sigma_i u_i(t)/N$ and $\Sigma_i v_i(t)/N$ may be treated as zero with negligible approximation error. Thus, summing both sides of (1) and (2) over i and dividing by N yields parallel equations, without shock terms, in the sample means $Y(t)$, $S(t)$, and $X(t)$. After using (5) and [from (3)] $X(t) = Y(t) - T(t)$ to substitute out $X(t)$, the parallel versions of (1) and (2) may be written

$$\text{(a)} \qquad \begin{pmatrix} Y(t+1) \\ S(t+1) \end{pmatrix} = A_0 + A \begin{pmatrix} Y(t) \\ S(t) \end{pmatrix}$$

where

$$A_0 = \begin{pmatrix} a+\mu(\beta-\gamma C)f(P) \\ \mu f(P) \end{pmatrix}, \quad A = \begin{pmatrix} (1-g)[\gamma+\mu(\beta-\gamma C)h(P)] & \delta+b\mu(\beta-\gamma C) \\ (1-g)\mu h(P) & \mu b \end{pmatrix}$$

and $\mu = 1/[1 + (C - P)h(P)]$. Equation (a) determines the time paths of $Y(t)$ and $S(t)$; the time paths of $T(t)$, $X(t)$, and $H(t)$ then follow [via (3) - (4) - (5)] from:

$$T(t) = gY(t) + (C - P)S(t+1), \qquad X(t) = Y(t) - T(t)$$

(b)

$$H(t) = [T(t) + G] / Y(t)$$

Turn now to the variance matrix of $[Y_i(t), S_i(t), X_i(t), T_i(t)]$. (1) and (2), after using (3) and (4) to substitute out $X_i(t)$ and $T_i(t)$, may be written

(c) $$\begin{pmatrix} Y_i(t+1) \\ S_i(t+1) \end{pmatrix} = B_O + B(t)\begin{pmatrix} Y_i(t) \\ S_i(t) \end{pmatrix} + U_i(t)$$

where

$$B_O = \begin{pmatrix} a + \gamma G + (\beta - \gamma P)[f(P) + h(P)G] \\ f(P) + h(P)G \end{pmatrix}, \quad U_i(t) = \begin{pmatrix} u_i(t) + (\beta - \gamma P)v_i(t) \\ v_i(t) \end{pmatrix}$$

$$\text{var}[U_i(t)] = V = \begin{pmatrix} \sigma_{uu} + (\beta - \gamma P)^2 \sigma_{vv} + 2(\beta - \gamma P)\sigma_{uv} & (\beta - \gamma P)\sigma_{vv} + \sigma_{uv} \\ (\beta - \gamma P)\sigma_{vv} + \sigma_{uv} & \sigma_{vv} \end{pmatrix}$$

$$B(t) = \begin{pmatrix} [\gamma + (\beta - \gamma P)h(P)][1 - H(t)] & \delta + (\beta - \gamma P)b \\ h(P)[1 - H(t)] & b \end{pmatrix}$$

Taking the variance matrix of both sides of (c) yields

(d) $$\Pi_O(t+1) = B(t)\Pi_O(t)B(t)' + V$$

where $\Pi_O(t)$ denotes the (2,2) variance matrix of $[Y_i(t), S_i(t)]$. Since $B(t)$ depends on time only through $H(t)$, whose time path is known from (a) - (b), then (a) - (b) - (d) determines the time path of $\Pi_O(t)$. To expand the matrix $\Pi_O(t)$ to the (4,4) variance matrix, call it $\Pi(t)$, of $[Y_i(t), S_i(t), X_i(t), T_i(t)]$, note from (3) and (4) that

$$\begin{pmatrix} X_i(t) \\ T_i(t) \end{pmatrix} = \begin{pmatrix} G \\ -G \end{pmatrix} + C(t)\begin{pmatrix} Y_i(t) \\ S_i(t) \end{pmatrix}, \text{ where } C(t) = \begin{pmatrix} 1 - H(t) & 0 \\ H(t) & 0 \end{pmatrix}$$

It follows that

(e) $$\Pi(t) = \begin{pmatrix} \Pi_O(t) & \Pi_O(t)C(t)' \\ C(t)\Pi_O(t) & C(t)\Pi_O(t)C(t)' \end{pmatrix}$$

Given the means and variance matrix, the coefficients of variation $[V_Y(t), V_S(t), V_X(t), V_T(t)]$, contemporaneous correlation matrix [call it $R(t)$], and one-period lag (parent-child) correlation matrix [call it $R_+(t)$] follow. Let $D(t) = \text{diag}[D_Y(t), D_S(t), D_X(t), D_T(t)]$ be the diagonal matrix whose diagonals are the square roots of the diagonals of $\Pi(t)$. Then

$$V_Y(t) = D_Y(t)/Y(t), \qquad V_S(t) = D_S(t)/S(t),$$

$$V_X(t) = D_X(t)/X(t),\ V_T(t) = D_T(t)/T(t),\ R(t) = D(t)^{-1}\Pi(t)D(t)^{-1},$$

$$R_+(t) = D(t)^{-1}\begin{pmatrix} B(t-1) & 0 \\ C(t)B(t-1) & 0 \end{pmatrix}\Pi(t-1)D(t-1)^{-1}$$

The text magnitudes $r_{YS}(t)$ and $r_{+SY}(t)$ denote the (1,2)nd element of $R(t)$ and (2,1)st element of $R_+(t)$, respectively; and the text magnitudes $[r_{+YY}(t), r_{+SS}(t), r_{+XX}(t), r_{+TT}(t)]$ are the diagonals of $R_+(t)$.

Finally, consider equilibrium and stability. It will be assumed that both eigenvalues of the matrix A have moduli less than one. It then follows from (a) and (b) that $[Y(t), S(t), X(t), T(t), H(t)]$ have unique and globally stable equilibrium values. They can be easily expressed as functions of parameters. As $H(t)$ converges to its equilibrium, the matrices $B(t)$ and $C(t)$ will converge to equilibrium values. It will be assumed that both eigenvalues of the equilibrium value of $B(t)$ have moduli less than one. It follows (see the mathematical discussion in the appendix to [5]) that $\Pi_O(t)$ in (d) and $\Pi(t)$ in (e) have unique and globally stable equilibrium values. They can be easily expressed as functions of parameters (again see the appendix to [5]).

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